

**Estimating FNSB Home Heating Elasticities of demand using the
Proportionally-Calibrated Almost Idea Demand System (PCAIDS)
Model: Postcard Data Analysis**

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Summary of Results

Findings indicate that overall household heating energy use decreased by 15% in 2016 relative to the same household level use in 2014/2015. Wood use (in cords) use decreased by approximately 32% in 2016 relative to 2014, and oil use (in gallons) decreased by approximately 9.64%. Change in overall household heating efficiency,¹ as well as differences in the severity of winter temperatures accounts for a portion of the decrease in household energy usage.² Results indicate that median estimates for own-price elasticities for oil are -0.274 and -0.353 respectively. Based on the predicted median values a 1% increase in the price of heating oil (mmBTU) is estimated to result in a reduction of 0.274% to 0.353% in the quantity (mmBTU) of residential heating oil consumed by the average household. As an increase in heating oil price is predicted to increase the use of firewood these predicted cross-price elasticities indicate heating oil and firewood are treated as substitutes. Based on robust regression the median cross-price elasticity of firewood with respect to a change in the price of heating oil is wood is 0.224.³ Based on median predictions, a 1% increase in the price of heating oil (mmBTU) is estimated to increase the consumption of wood (mmBTU) approximately 0.224%.

Additionally, given a 10% increase in the price of oil and average firewood use in FNSB households of 1.68 cords (51.29 mmBTU) annually and an estimated cross-price of 0.224, a 10% increase in the price of oil translates into an additional 0.03 cords or (1.14 mmBTU) burned per household. The resulting confidence intervals for the means of the own- and cross-price

¹ On average, wood-burning devices have lower heating efficiency than oil heating devices. Thus, a relative shift from wood to oil will result in a decrease in fuel energy needed. Changes in overall household heating efficiency due to the shift in wood vs. oil use¹ account for 0.7% of the decrease in energy usage (Sierra Research Inc).

² Differences in the severity of winter temperatures in 2016 versus 2014 and 2015 account for 3.8% of this 15% decrease (Sierra Research Inc).

³ Both estimates are found using a robust regression which uses a weighted estimation scheme to control for leverage exerted by potential outliers.

elasticities, at 95% confidence level, validate that the resulting coefficients for elasticity of demand and the elasticity measurements are statistically significant.

Introduction

In December of 2009, the EPA designated Fairbanks as a Serious Nonattainment Area for Particulate Matter (PM)_{2.5} emissions for the 2006 24-hour air quality standards. The Fairbanks North Star Borough (FNSB) has recorded some of the highest levels of PM_{2.5} in the United States. The largest contributors to PM_{2.5} in the FNSB are wood stoves and hydronic heaters.⁴ Currently, two of the measures implemented to mitigate PM_{2.5} emissions are requiring a removal of inefficient wood heating devices when a property is sold or leased⁵ and requiring commercial wood sellers to register with the state and report the moisture content of wood they are selling to residential wood-burners.⁶

According to the American Lung Association - based on PM_{2.5} emissions data Fairbanks ranked No. 5 in its 2016, 2017 and No. 4 in its 2018 State of the Air Report for People at Risk during Short-Term 24-hour PM_{2.5} episodes (State of the Air, 2016, 2017, 2018). Only four areas – all located in California ranked worse than Fairbanks during this period.

Analysis in this paper is focused on using community level household energy consumption data and prices to determine the own-price elasticities of oil and cross-price elasticities of firewood with respect to changes in the price of home heating oil from 2016-2018. Price elasticities of energy demand have become increasingly relevant in determining the economic and environmental effects of energy policies on countries and communities alike. Elasticity values can be used to help

⁴ (U.S. Environmental Protection Agency, n.d.)

⁵ Alaska State regulation. 18 AAC 50.077 and 18 AAC 50.079

⁶ Alaska State regulation. 18 AAC 50.076(d)

identify how residents of the FNSB will alter home heating preferences in response to a change in the price of heating oil. Of particular interest, given the need to improve local air quality, is how firewood usage might change if the price of home heating oil increases if the use of lower sulfur fuels (i.e., #1 heating oil and ultra-low sulfur diesel) is mandated. The analysis draws on the “proportionally calibrated almost ideal demand system” (PCAIDS) developed by Epstein and Rubinfeld (2002) and also presented by Coloma (2006) to estimate the own- and cross-price elasticities of demand for home heating oil and firewood.

Own-price and cross-price elasticities were estimated through an application of data from the 2016 Fairbanks Home Heating Household Survey, a postcard-type instrument that provided a streamlined approach to collecting information on residential home heating practices. The survey collected more specific data on wood and heating oil usage from the same set of households in the Fairbanks North Star Borough (FNSB) during the period 2014-2016. The panel dataset provides two observations for each household surveyed making it possible to control for heterogeneity across responding units. The postcard panel data is used to estimate own-price elasticities of oil and cross-price elasticities of firewood with respect to changes in the price of home heating oil from 2014 to 2016.

Own- and Cross- Price Elasticity of Demand

Own-price elasticity of demand measures how sensitive the quantity demanded of a good or service is to a change in price. The sensitivity of the quantity of heating oil consumed by a household relative to changes in fuel price depends on several factors, including: temperature preferences, heating appliance(s) type and efficiency, the presence of alternate heating appliances in the home, home age, and overall energy efficiency of the home. Demand is said to be “inelastic”

when the percentage change in quantity demanded is less than the percentage change in price. Demand is said to be “elastic” when the percentage change in quantity demanded is greater than the percentage change in price.

Cross-price elasticity of demand estimates the responsiveness in the quantity demanded of one good given the change in price of another good. In this case, we are looking at the quantity demanded of firewood given a change in the price of heating oil. When the cross-price elasticity of demand is positive, the goods are substitutes. In the case of substitutes, as the price of one good increases, consumers can substitute with the relatively less expensive good. Meaning that, all else equal, as the price of the good increases demand for the corresponding substitute good increases. Alternatively, when the cross-price elasticity of demand is negative, the goods are complements. As the price of one good increases, the demand for both goods will decrease or vice versa.

Estimates of the own-price elasticity of demand for heating oil will be influenced by the presence of an alternate heating source, in this instance a wood stove or wood stove insert. Based on standard economic theory homes without an alternate source of heat will have a more inelastic demand for home heating oil. Conversely, homes with an additional source of heat, such as a wood stove or insert would be expected to be less sensitive to heating oil price changes since they will be able to shift a portion of home heating needs to the other appliance. The estimated cross-price elasticity of firewood demand in response to a change in heating oil price measures the corresponding increase in firewood consumption.

For example, if the cross-price elasticity of wood is 0.5, a 1% decrease in the price of oil will decrease firewood consumption by 0.5% as households substitute use towards oil given the lower relative price. It is assumed that wood and oil are substitute goods – or as the price of oil decreases, households tend to increase oil consumption and decrease wood consumption.

Increases in oil prices would increase firewood consumption and subsequently increase PM2.5 emissions in the non-attainment zone.

PCAIDS Model

Own and cross-price elasticities for residential heating oil demand and firewood consumption were estimated using the proportionally calibrated almost ideal demand system (PCAIDS). In many instances data limitations make it difficult to estimate the full almost-ideal demand system (AIDS) model developed by Deaton and Muelbauer (1980). An alternative to the AIDS model is the “proportionally calibrated almost ideal demand system” (PCAIDS) model developed by Epstein and Rubinfeld (2002).

The PCAIDS model has fewer data requirements than the typical AIDS model, providing an alternative strategy for estimating demand systems in the presence of imperfect information. The PCAIDS model avoids many of the challenges of the traditional AIDS framework, notably the estimation of a large set of parameters and the potential for low statistical significance, implausible magnitudes, or wrong signs inconsistent with economic theory. The PCAIDS model applies the same logic as the AIDS model, but incorporates restrictions to make all elasticity values depend on a single parameter and market shares of the respective goods. The restrictions imposed ensure the correct signs and magnitudes of required parameters and elasticities (Epstein & Rubinfeld, 2002). The modeling approach used here follows Coloma (2006) who presents a two stage process of deriving own- and cross-price elasticities of demand using the PCAIDS framework.

Household expenditure shares for residential heating oil (S_o) and firewood (S_w) measure the proportional share of the home heating budget spent on each heating fuel type. Total household expenditures on wood and oil in mmBTU are calculated as:

$$E_T = (E_O + E_W) \quad (1);$$

Where E_O is the household expenditure on heating oil, E_W is household expenditures on wood, and E_T total household expenditures on oil and wood. The expenditure shares of oil and wood can then be calculated directly:

$$S_o = \frac{E_O}{E_T} \quad (2);$$

$$S_w = \frac{E_W}{E_T} \quad (3);$$

$$S_T = S_o + S_w \quad (4);$$

Where S_1 is the share of household expenditures on oil, S_2 the share of household expenditures on wood.

It is important to note that approximately 31% of households surveyed indicate collecting firewood for use. For present purposes, it is assumed that the time and input costs associated with the collection of firewood are commensurate with the market price used in the analysis. The dependent shares are modeled as a function of the relative fuel price ratio and other factors (Y) which include the square footage of the home, age of the home in years, the elevation at the housing location, and zip code level median household income. A year level fixed effect controls for annual variations due to changes in heating degree days. Following Coloma (2006) three separate equations are estimated in order to calculate the appropriate elasticities.

In order to estimate the own-price elasticity of demand for oil and cross-price elasticity of demand for wood, two required parameters must be recovered from the models: a_{11} is represented as the adding-up property of the PCAIDS model which is equal to the summation of the cross price parameter (Coloma 2006), and n the aggregate demand elasticity of oil. Using available price data for wood and oil, and expenditure shares of wood and oil, the dependent shares model can be applied to gain a direct estimate for a_{11} . Following Coloma (2006) the demand system models are derived:

$$\frac{s_o \cdot (1 - s_o)}{s_w} = -a_i \cdot b_{10} + a_{11} \cdot \ln\left(\frac{p_o}{p_w}\right) - a_{11} \cdot b_{1Y} \cdot Y \quad (5);$$

Where $\ln\left(\frac{p_o}{p_w}\right)$ is the natural log of the relative price ratio of oil and wood per million of BTU (mmBTU). By estimating a_{11} through equation (1) as the coefficient of $\ln\left(\frac{p_o}{p_w}\right)$, the required a_{11} parameter assumed by the PCAIDS model can be recovered. The parameter a_{11} is of interest as it helps describe the relative spending behavior of price-taking buyers and is a required input to calculate both own- and cross-price elasticities. Similarly, the own-price elasticity of demand for wood a_{22} and a_{12} can be calculated by estimating the dependent shares model for wood:

$$\frac{s_w \cdot (1 - s_w)}{s_o} = -a_i \cdot b_{10} + a_{22} \cdot \ln\left(\frac{p_w}{p_o}\right) - a_{22} \cdot b_{1Y} \cdot Y \quad (6);$$

Where $\ln\left(\frac{p_w}{p_o}\right)$ is the natural log of the relative price ratio of wood and oil respectively in mmBTUs. Parameter a_{22} , the own-price elasticity of wood can be recovered as the coefficient of $\ln\left(\frac{p_w}{p_o}\right)$ which can then be used to estimate the cross-price effect of wood with respect to a change in price of heating oil.

In the second stage of the model, the household heating demand equation is estimated to determine aggregate demand elasticity (n):

$$\ln(Q) = C_0 + n \cdot \ln(P_A) + C_Y \cdot Y \quad (7);$$

Where Q is the level of mmBTU consumption for the household, $\ln(P_A)$ is a natural log of weighted average price per mmBTU for wood and oil, and C represents the estimated coefficients. The required parameter n is recovered as the coefficient of $\ln(P_A)$. Parameter $\ln(P_A)$ is expected to have a negative coefficient to satisfy the law of demand in the aggregate demand equation.

The aggregate mmBTU consumption (Q) of home heating fuel is calculated as follows:

$$Q_G = \frac{G}{P_1} \quad (8);$$

$$Q_W = \frac{W}{P_2} \quad (9);$$

$$Q = Q_G + Q_W \quad (10);$$

Where G represents the quantity of oil in mmBTUs consumed, W represents the quantity of wood consumed in mmBTUs by household, and P_O and P_W are the price per mmBTU of oil and wood respectively; Q_G is the aggregate mmBTU consumption of gallons of heating oil, Q_W is the aggregate mmBTU consumption of wood, and Q represents the aggregate product quantity in mmBTUs consumed by household.

The weighted average price per mmBTU of wood and oil is calculated by adjusting the wood and heating oil prices by the respective household spending shares; the adjusted values are then summed:

$$P_{AO} = AP_1 \cdot S_1 \quad (11);$$

$$P_{AW} = AP_2 \cdot S_2 \quad (12);$$

$$P_A = P_{AO} + P_{AW} \quad (13);$$

Where P_{AO} is the weighted average price per BTU of oil, P_{AW} is the weighted average price per BTU of wood, AP_1 is the average price per BTU of oil, AP_2 is the average price per BTU of wood, and P_A , is the weighted average price per mmBTU consumed by the household.

Using the weighted average prices per mmBTU, the aggregate demand equation can be estimated to recover the required parameter n , which the coefficient of $\ln(P_A)$. The recovered parameters of n and a_{11} can be used to calculate a_{22} , a_{12} and the own-price (n_{own}) and cross-price (n_{cross}), elasticities of demand for oil and wood respectively.

Using the n and a_{11} parameters, the own- and cross-price elasticity be calculated directly as follows:

$$n_{own} = -1 + \frac{a_{11}}{S_O} + S_O \cdot (n + 1) \quad (14);$$

$$n_{cross} = \frac{a_{12}}{S_O} + S_W \cdot (n + 1) \quad (15);$$

Where n is the aggregate demand elasticity of the product recovered from equation (2), n_{own} is the own-price elasticity of oil and n_{cross} is the cross-price elasticity of demand for wood. The cross-price elasticity of demand, a_{12} and a_{22} , is calculated with respect to the expenditure shares, and the a_{11} parameter estimated in equation (1). Estimating the a_{12} cross-price parameters has the following relationship with the own-price parameters of the second product (wood) a_{22} from equation (2):

$$a_{22} = \frac{S_W \cdot (1 - S_W)}{S_2 \cdot (1 - S_O)} \cdot a_{11} \quad (16);$$

$$a_{12} = \frac{-S_o}{(1-S_W)} \cdot a_{22} \quad (17);$$

a_{12} is then used to estimate n_{cross} , the cross-price elasticity of demand for wood with respect to a change in oil price.

Data and Analysis

The 2016 home heating postcard survey was conducted by Sierra Research and consisted of questions which asked respondents about their household's annual use of home heating oil and firewood (Sierra, 2016). A total of 1,401 postcards was mailed, encompassing all the respondents in the 2014 and 2015 home heating telephone surveys and providing pre-printed 2014 or 2015 device/usage data for each individual respondent. A total of 271 postcards was ultimately returned over the ensuing three months, reflecting a return rate of just under 20%.

The set of 271 responding households provided heating fuel use information in either 2014 or 2015 by telephone survey. Data from the 2014/2015 telephone survey and 2016 postcard survey were paired by household. Sierra Research performed a series of calculations to validate the data for each household. Sierra calculated fuel use data by device from each survey "point" (2014/2015 vs. 2016 postcard) which were then translated into estimates of winter heating energy use, measured in BTUs.

To ensure the validity of the household responses, Sierra Research looked at total household energy use in BTUs and compared the results based on the 2014/2015 data point and that from the 2016 postcard survey. If the energy use from one survey was dramatically different from the other, both data points for the household were deemed invalid. Sierra Research utilized a validation threshold of a $\pm 75\%$ change in energy use to validate or reject the data for each

household.⁷ Through the validation process, 38 out of the 271 respondents were deemed “invalid.” All models are estimated using the 233 responses determined to be valid by Sierra Research.

Information on the square footage and age of respondent homes were collected by Sierra Research. Data on the median household income by zip code was collected from the American Community Survey (ACS). Home size as well as home age have been shown to be important explanatory factors in home heating demand. Likewise, household income is a standard variable included in home heating demand models (Rehdanz & Meier, 2008), (Sardianou, 2007) and (Song, Aguilar, Shifley, & Goerndt, 2012).

Rehdanz & Meier (2008) examine determinants of heating expenditures which include socio-economic and building characteristics and analyze households’ heating behavior from 1991 to 2005. The regression controls for annual household income, household size, average age of householder, and employment of householder. Sardianou (2007) investigates the determinants of household energy conservation patterns in Greece employing a cross-sectional data using monthly income, number of rooms in the dwelling, and dwelling size. Song et al. (2012) examines the factors affecting individual U.S. household wood energy consumption. The regression controls for number of household members, household consumption of wood, location, household income, household size in square meters, annual heating degree days, and price of wood.

Song et al. (2012) estimates that for every 1% increase in non-wood energy prices is predicted to induce a 1.55% increase in firewood energy consumption.

⁷ Sierra Research indicated that the validation level was selected to account for the combination of variations due to reporting precision of wood use, year-to-year differences in winter severity, and effects of differences in net heating efficiencies across the key devices.

Average household heating oil and firewood use in gallons, cords, and mmBTU from 2014/2015 to 2016 are presented in *Table 1*.

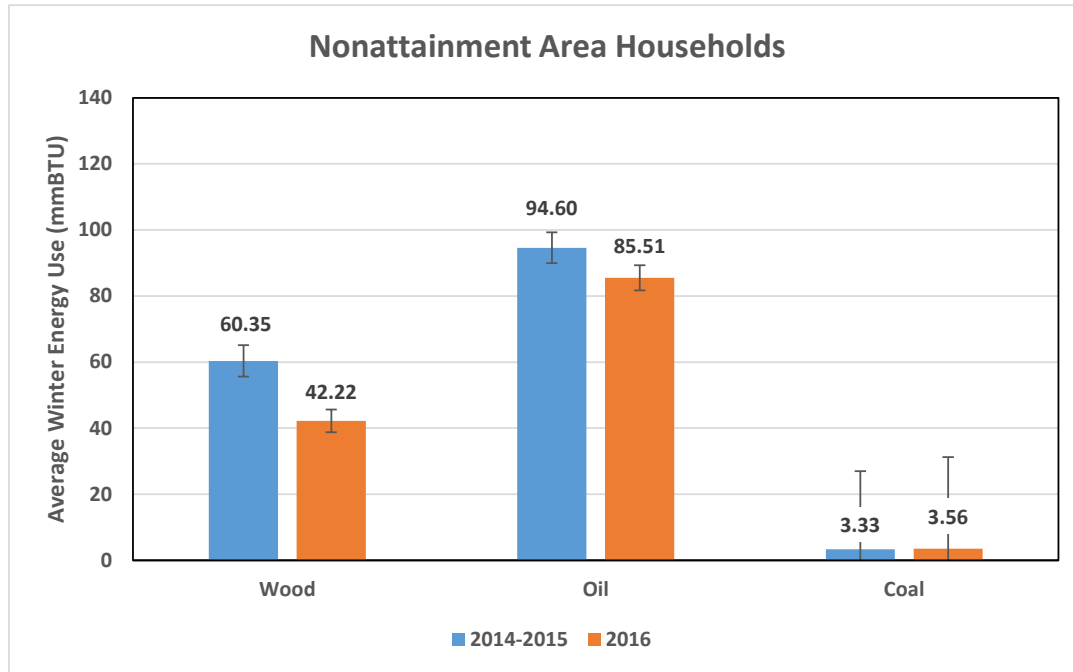
Table 1: Average Household Use by Fuel Type				
Year	Central Oil (gallons)	Wood (cords)	Central Oil (mmBTU)	Wood (mmBTU)
2014/2015	658.62 (535.65)	2 (2.71)	94.60 (72.31)	60.35 (56.41)
2016	595.07 (451.36)	1.36 (1.85)	85.51 (60.93)	42.22 (40.93)
Total	626.84 (495.65)	1.68 (2.34)	90.05 (66.91)	51.29 (49.66)

Note: Standard errors in parentheses.

There was a notable decrease in average reported household heating oil and firewood usage between 2014/2015 and 2016 (*Table 1*). The change in heating oil use of 658.62 gallons in 2014/2015 to 595.07 gallons in 2016 or in terms of mmBTU, 94.60 to 85.51 mmBTU, is approximately 9.6%. The change from the household average use of 2 cords to 1.36 cords annually represents a 32% decrease in wood use from 2014/2015 to 2016. The associated reduction in terms of mmBTU's, is 60.35 mmBTU to 42.22 mmBTU.

Figure 1 provides a comparison of average winter season household energy use by fuel type between the 2014/2015 and 2016 surveys. This is visually represented in *Table 1*.

Figure 1:
Change in Average Winter Household Energy Use (mmBTU) by Fuel Type,
Entire Nonattainment Area



Source: Sierra Research Inc, Postcard Data White Paper, 2016

Table 2: Total Household Energy Consumption in (millions) of BTUs and Heating Degree Days

Year	Energy consumption (mmBTU)	Heating Degree Days
2014/2015	134.42 (111.91)	10,199
2016	119.14 (78.75)	9,735
Total	12.78 (96.97)	9,967

Note: Standard deviations in parentheses

These reductions are driven by decreases in the number of heating degree days between 2014/2015 and 2016. *Table 2* displays household change in total energy use in mmBTU, and Heating Degree Days from 2014/2015 to 2016. The change from 138.81 mmBTU to 118.1

mmBTU annually represents a 15% decrease in energy usage from 2014/2015 to 2016. The change from 10,119 to 9,735 heating degree days represents a 3.75% decrease in annual heating degree days in the FNSB.

Table 3: Market Prices (Wood and Oil in Dollars)

Year	Oil Price	Wood Price	Oil Price (mmBTU)	Wood Price (mmBTU)	HH Expenditures	Oil Shares	Wood Shares
2014/2015	3.38	275.51	24.87	13.53	\$2,900 (\$2,985)	0.71 (0.36)	0.29 (0.36)
2016	2.39	266.99	17.70	13.11	\$2,600 (\$1,778)	0.76 (0.30)	0.24 (0.30)
Net Change	0.99	8.52	7.17	0.42	\$300 (\$1,207)	-0.05 (0.06)	0.05 (0.06)
Average	2.87	271.25	21.04	13.32	\$2,750 (\$2,459)	0.74 (0.33)	0.26 (0.33)

Note: Standard deviations in parentheses

Table 3 presents the change in market prices for wood and oil from 2014/2015 to 2016 in gallons and cords of wood, as well as in price per mmBTU by fuel type.⁸ The change from \$275.51 to \$266.99 represents a 3.10% decrease in the market price for a cord of wood. The change from \$3.38 to \$2.39 represents a 28.66% decrease in the market price for heating oil. Overall expenditures for households decreased from \$2,900 to \$2,600 annually, representing a 10% decrease in annual household heating expenditures. Oil shares increase from 71% to 76% of household heating expenditures between the two time periods, this represents a 7% increase in expenditures on heating oil. Wood shares decreased from 29% to 24% of household heating expenditures, this represents a 17% decrease in household expenditures on firewood between the two time periods.

⁸ Firewood and Oil prices found from the Alaska Energy Data Gateway.

Table 4: Sociodemographic and Household Characteristics

	Median HH Income	Home size (Sqft)	Home age (years)	Elevation (feet)
Mean	\$73,984 (\$10,070)	1,960 (803)	34 (14)	620.84 (238.25)

Note: Standard deviation in parentheses

Table 4 represents median household income, average home size, average home age, and elevation in meters above sea-level in the data over all time periods.

Estimated Models and Results

The household share of expenditures devoted to firewood and heating oil, as well the total share of expenditures is estimated for the average household. Models in this analysis were specified emulating the empirical model employed by (Song, Aguilar, Shifley, & Goerndt, 2012) looking at household heating preferences and (Coloma, 2004).

Table 5: Mean Proportion of Appliance Type

	Central Oil	Direct Vent Oil	Wood Stove	Wood-Oil	Wood Collect
Mean	0.83 (0.37)	0.17 (0.38)	0.56 (0.49)	0.42 (0.49)	0.32 (0.47)

Note: Standard errors in parentheses

Table 5 presents a summary of reported appliance use in FNSB homes. 83% of households report using a central oil heating appliance, 17% of households report using a direct vent appliance, 56% of households report using cord-wood (primarily for wood stoves), and 42% of households report using a combination of both wood and oil appliances. The *woodcollect* variable indicates whether a household reports collecting or purchasing firewood – approximately 32% of households report collecting their own firewood in the dataset.

Table 6: Summary Statistics of Regression Variables

Year	Dependent Shares	Dependent Shares Wood	Aggregate Demand	Ln(P_a)
2014/2015	0.52 (0.32)	0.18 (0.24)	4.78 (0.51)	3.07 (0.28)
2016	0.59 (0.30)	0.20 (0.26)	4.61 (0.50)	2.81 (0.89)
Average	0.57 (0.31)	0.19 (0.25)	4.70 (0.51)	2.94 (0.25)

Note: Standard deviations in parentheses

Table 6 presents the summary statistics of regression variables used in the models below. *Dependent Shares* increased from 0.52 to 0.59 between the two time periods, as oil shares tended to increase between the two time periods, the dependent shares are expected to increase. *Ln(BTU_Ratio)* decreased given the ratio of the price per mmBTU decreased between the two time periods. Aggregate demand decreased between both time periods which could be contributed to a decrease in the overall heating degree days in the FNSB. Weighted price variable $\ln(P_a)$ also decreased given the weighted price per mmBTU dropped for both oil and firewood between the two periods.

Two estimation strategies were used, standard linear regression and robust regression. Robust regression is an alternative to least squares regression which uses a weighted estimation scheme to control for heteroscedasticity and leverage exerted by potential outliers in the data.⁹ Robust regression first runs the OLS regression and calculates Cook's distance for each

⁹ An observation with an extreme value on a predictor variable is a point with high leverage. Leverage is a measure of how far an independent variable deviates from its mean. High leverage points can have a great amount of effect on the estimate of regression coefficients (UCLA: Statistical Consulting Group, 2016).

observation and drops any observation with a Cook's distance greater than 1.^{10 11 12} In short, the most influential points are dropped, then those observations with large absolute residuals are weighed downward. Using the expenditure shares, the equation (5) is estimated as follows:

$$\begin{aligned} Depshares = & \beta_0 + \beta_1 \ln\left(\frac{P_o}{P_w}\right) + \beta_2 year + \beta_3 DV + \beta_4 CentralOil + \beta_5 Woodcollect + \\ & \beta_6 \ln(size) + \beta_7 \ln(MHH) + \beta_8 elevation + \beta_9 homeage + u \end{aligned} \quad (18);$$

Where $\ln\left(\frac{P_o}{P_w}\right)$ is the natural log of relative price ratio of both wood and oil in mmBTU, *DV* is a dummy variable indicating if a household has a direct vent appliance, *CentralOil* is a dummy variable indicating if a household has a central oil appliance, *Woodcollect* is a dummy variable indicating if the household bought or collected wood, *size* is the size of the home in square feet, *year* is a dummy variable indicating the survey year where the use was reported,¹³ *MHH* is the median household income by zip code,¹⁴ *elevation* is the meters the home is located above sea-level, and *homeage* is the age of the home in years. Regression results are displayed below.

¹⁰ Cook's distance measures the influence of the observation on the fitted values. It assigns leverage to variables based on their distance from the fitted values. See (Kutner, Nachtsheim, Neter, & William, 2005) for further information.

¹¹ For more information on robust regression techniques please see: Verardi, V., & Croux, C. (2009). Robust Regression in Stata. Stata Journal, 439-453.

¹² Using Stata defaults, robust regression is approximately 95% as efficient as OLS (Hamilton, 1991).

¹³ Dummy variable *year* also represents the heating degree days in the FNSB for that particular year. Heating degree days and year effect variables are not included, as they are perfectly correlated.

¹⁴ Median household income was collected from the American Community Survey from 2011-2015.

Table 7: Dependent Shares Model Results (Heating Oil)

VARIABLES	Linear Regression	Robust Regression
	Depshares	Depshares
ln(BTU_Price_ratio)	0.0290 (0.140)	-0.0128 (0.142)
2015	-0.0604 (0.0665)	-0.0883 (0.0672)
2016	0.00922 (0.0773)	-0.0351 (0.0781)
Direct Vent	0.161*** (0.0582)	0.172*** (0.0588)
CentralOil	0.477*** (0.0559)	0.538*** (0.0565)
Woodcollect	-0.0449 (0.0362)	-0.0474 (0.0366)
ln(size)	-0.0958* (0.0508)	-0.0980* (0.0514)
ln(MHH)	0.220 (0.161)	0.141 (0.163)
Elevation	-0.000102 (7.51e-05)	-0.000114 (7.59e-05)
Homeage	-0.000720 (0.00141)	-0.000822 (0.00143)
Constant	-1.458 (1.832)	-0.526 (1.852)
Observations	244	244
R-squared	0.285	0.333

Note: Standard errors in parentheses
 *** p<0.01, ** p<0.05, * p<0.1

Dummy variables *DirectVent* and *CentralOil* are statistically significant at the 1% level, *size* is statistically significant at the 10% level in both the linear and robust regression models. *Size* is statistically significant at the 10% level in both models. 244 observations are analyzed in the heating oil dependent shares model, this is due to the way the dependent shares heating oil model is calculated. If a household does not have an expenditure share on wood, there would be no dependent shares calculated for that household given the denominator (wood share) would be zero, which would generate a missing dependent shares observation for that household.

To estimate the cross-price effect of wood with respect to a change in oil, equation (2) is estimated using:

$$DepSharesWood = \beta_0 + \beta_1 \ln\left(\frac{P_W}{P_O}\right) + \beta_2 year + \beta_3 DV + \beta_4 CentralOil + \beta_5 Woodcollect + \beta_6 \ln(size) + \beta_7 \ln(MHH) + \beta_8 elevation + \beta_9 homeage + u \quad (19);$$

Table 8: Dependent Shares Wood Regression Results

VARIABLES	Linear Regression	Robust Regression
	Dependent Share Wood	Dependent Share Wood
ln(BTU_Ratio_Wood)	-0.00764 (0.0354)	-0.0281 (0.0190)
2015	0.0331 (0.0433)	-0.00891 (0.0233)
2016	0.0373 (0.0302)	-0.00646 (0.0162)
Direct Vent	-0.00407 (0.0504)	-0.0690** (0.0271)
CentralOil	-0.253*** (0.0539)	-0.126*** (0.0290)
Woodcollect	0.207*** (0.0242)	0.198*** (0.0130)
ln(size)	0.0139 (0.0272)	-0.00558 (0.0146)
ln(MHH)	0.0608 (0.0863)	0.0146 (0.0464)
Elevation	4.83e-05 (5.38e-05)	1.83e-05 (2.89e-05)
Homeage	-0.000833 (0.000866)	-0.00117** (0.000466)
Constant	-0.484 (0.976)	0.0368 (0.525)
Observations	407	407
R-squared	0.288	0.422

Note: Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Dummy variables *Central Oil* and *Woodcollect* are statistically significant at the 1% level a lower in both the robust and linear regressions. *Direct Vent* and *Homage* were statistically significant at the 1% and 5% levels respectively in the robust regression. 407 observations are

analyzed in the wood dependent shares model, again the discrepancy between the number of valid households and the number of analyzed households is due to the way the wood dependent shares model is calculated. If a household does not have an expenditure share on oil, there would be no dependent shares calculated for that household given the denominator (oil share) would be zero, which would generate a missing wood dependent shares observation for that household.¹⁵

The aggregate market demand model is estimated using equation (3):

$$\ln(Q) = \beta_0 + \beta_1 \ln(P_A) + \beta_2 year + \beta_3 DirectVent + \beta_4 CentralOil + \beta_5 Woodcollect + \beta_6 \ln(size) + \beta_7 \ln(MHH) + \beta_8 elevation + \beta_9 homeage + u \quad (20);$$

Where Q is the aggregate household heating consumption in mmBTU and $\ln(P_A)$ is the natural log of the weighted average mmBTU prices of firewood and heating oil. Parameter $\ln(P_A)$ is expected to have a negative coefficient representing the inverse relationship between price and quantity demanded. All other variables in the model are the same as described in the dependent shares model. Regression results are displayed below.

¹⁵ The odd number of households analyzed in the dependent shares wood model is due to some households reporting oil use in one time period and reporting no oil use in the other time period. Therefore, the household would have a value for wood dependent shares in one time period, but not in the other.

Table 9: Aggregate Demand Model Results

VARIABLES	Linear Regression ln(Aggregate Demand)	Robust Regression ln(Aggregate Demand)
Ln(P_a)	-0.266** (0.109)	-0.301*** (0.111)
2015	-0.0329 (0.0596)	-0.0489 (0.0608)
2016	-0.256*** (0.0618)	-0.268*** (0.0630)
Direct Vent	0.127* (0.0769)	0.119 (0.0784)
Central Oil	0.515*** (0.0766)	0.513*** (0.0781)
Wood Collect	0.114** (0.0446)	0.109** (0.0455)
ln(size)	0.443*** (0.0499)	0.438*** (0.0509)
ln(MHH)	0.175 (0.154)	0.169 (0.158)
Elevation	-0.000179* (9.29e-05)	-0.000185* (9.47e-05)
Homeage	0.000830 (0.00157)	0.000678 (0.00160)
Constant	-0.0412 (1.781)	0.176 (1.818)
Observations	434	434
R-squared	0.383	0.371

Note: Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Variables $\ln(P_a)$, 2016, *CentralOil* and $\ln(size)$ were statistically significant at the 1% level for both linear and robust regressions. *Elevation* and *Woodcollect* are statistically significant at the 1% and 5% level respectively, for both linear and robust regressions. *DirectVent* is statistically significant at the 10% level for the linear regression only.

It is important to see a negative coefficient on $\ln(P_a)$ to represent the inverse relationship between price and quantity demanded. The positive coefficient on the $\ln(size)$ indicates that as size of the home increases, there will be an increase in the aggregate demand for home heating, which is expected. In this case, a 1% increase in *size* results in a 0.443% increase in demand.

Estimated mean and median own- and cross-price elasticities for the sample are presented in Table 6 below.

Table 10: PCAIDS Estimates of Own- and Cross-Price Elasticity Estimates for Residential Heating Oil and Firewood

	Linear Regression	Robust Regression
Own-Price Oil (Mean)	-0.351 [-0.365, -0.342]	-0.455 [-0.475, -0.437]
Own-Price Oil (Median)	-0.274	-0.353
Cross-Price Wood (Mean)	0.259 [0.238, 0.288]	0.260 [0.242, 0.287]
Cross-Price Wood (Median)	0.219	0.224

Note: Confidence intervals are in brackets

Table 10 represents the own- and cross-price elasticity estimates for residential heating oil and firewood using the PCAIDS model. Estimated own-price elasticities indicate that residential heating oil demand is relatively insensitive to price changes over the observation period (2014-2016). Based on the predicted median values a 1% increase in the price of heating oil (mmBTU) is estimated to result in a reduction of 0.274% to 0.353% in the quantity (mmBTU) of residential heating oil consumed by the average household. Likewise, an increase in heating oil price is predicted to increase the use of firewood since the predicted cross-price elasticities indicate heating oil and firewood are treated as substitutes. Based on the median value, a 1% increase in the price of heating oil (mmBTU) is estimated to increase the consumption of wood (mmBTU) from 0.219% to 0.224%.

Using the average firewood use in the FNSB of 1.68 cords (51.29 mmBTU) from 2014-2016, and the estimated median cross-price elasticity of wood of 0.224, given a hypothetical 10% increase in oil price is estimated to increase the consumption of wood by 2.24%. This translates

into an estimated additional use of 0.03 cords or 1.14 (mmBTU) given a 10% increase in the price of oil. Confidence intervals were constructed for the means of the own- and cross-price elasticities at 95% confidence level, it can be inferred from a statistically significant aggregate demand elasticity coefficient. The confidence intervals do not contain zero in either the own- or cross-price elasticity of demand, indicating that both elasticity measurements can be assumed to be statistically significant.

Table 11: Oil Price Elasticity Estimates in Literature

Author(s)	Own-price elasticity of Oil
Alberini, Gans, & Velez-Lopez, (2011)	-0.556 to -0.65
Galvin & Blank-Sunikka (2012)	-0.39 to -0.47
Madlener, Bernstein & Gonzalez (2011)	-0.15 to -0.34

Table 11 represents the results of other empirical results on own-price elasticity of demand for residential heating oil. Alberini, Gans, and Velez-Lopez (2011) estimate the own-price elasticity to be between -0.556 to -0.65. Galvin and Blank Sunikka (2012) estimate the own-price of heating oil to be between -0.39 to -0.47. Madlener, Bernstein & Gonzalez (2011) estimate the own-price elasticity to be between -0.15 to -0.34. The wide range of own-price elasticity measurements is due to difference in specification of the models, location, household preferences in that location, time-period of the dataset, etc. Estimates from this analysis fall within the range of other peer-reviewed journal articles. Due to the lack of peer-reviewed journal articles, Song et al., (2012) estimates the only cross-price elasticity of wood with respect to other non-wood energy prices, obtaining a cross-price elasticity of demand of 1.55.

Limitations

A few considerations should be mentioned. First, because the share equations were estimated separately homogeneity and symmetry restrictions were not imposed on the demand system. Second, given that the estimated elasticities are not normally distributed the median values presented in *Table 6* provide a better measure of central tendency in the data. Third, while estimates across both robust and linear models are similar, those produced using the robust model address the leverage exerted by outliers. This consideration is important when noting potential issues associated with recall and accuracy in the post card survey data. Finally, many households in Fairbanks report collecting their own wood instead purchasing. However, data on the time-value of money, or the length of time spent collecting wood is not available for this dataset. This forces our analysis to assume that the time-value of their money is equal to the market price of wood, which may not always be the case.

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